Errata in “Statistical Mechanics in a Nutshell”

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General observation

The definition of **Convexity** given in chap. 2, sec. 2.4, is the opposite of that commonly used in mathematics. Since it is used consistently along the book, I do not correct the several cases in which the expressions in the book are at variance with common usage. I plan to change the expression in a futhre edition of the book.

Chapter 2. Thermodynamics

2.17 Equations of state

- `Page 40, line 12 from bottom. Read:`
  obtained by deriving
  `correct to:`
  obtained by differentiating

Chapter 3. The Fundamental Postulate

3.1 Phase Space

- `Page 56. Line 7. Eq. (3.1). Read:`

$$H = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + U(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) \right].$$

  `correct to:`

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + U(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N).$$

3.5 Quantum States

- `Page 66, line 2. Read:`
  moving in one dimension along a segment of length n.
  `correct to:`
  moving in one dimension along a line of length L.

- Page 66, line 3. Read:
  possible energy values are $E_n = \hbar^2\pi^2 n^2/L^2$
  correct to:
  possible energy values are $E_n = \hbar^2\pi^2 n^2/(2mL^2)$

3.14. The p–T Ensemble

- Page 81, line 18 from bottom. Eq. (3.111) 2nd line. Read:

$$= k_{\mathrm{B}}T^2 \left.\frac{\partial V}{\partial T}\right)_p = k_{\mathrm{B}}T \left.\frac{\partial E}{\partial p}\right)_T .$$

correct to:

$$= -k_{\mathrm{B}}T \left.\frac{\partial E}{\partial p}\right)_T = k_{\mathrm{B}}T^2 \left[\left.\frac{\partial V}{\partial T}\right)_p + \frac{p}{T} \left.\frac{\partial V}{\partial p}\right)_T \right].$$

3.18 Fluctuations of Uncorrelated Particles

- Page 88. From the end of line 4 to the end of the section. Read:
  In fact, one has...
  ...Therefore,

$$p = \frac{k_{\mathrm{B}}T}{v} = \frac{Nk_{\mathrm{B}}T}{V}. \tag{3.153}$$

correct to:
In fact, one has

$$\left\langle N^2 \right\rangle - \left\langle N \right\rangle^2 = \left.\frac{\partial^2 \ln Z_{\mathrm{GC}}}{\partial(\mu/k_{\mathrm{B}}T)^2}\right)_{T,V} = k_{\mathrm{B}}T \left.\frac{\partial N}{\partial \mu}\right)_{T,V}. \tag{3.150}$$

On the other hand, since μ is an intensive variable, function of T and of the extensive variables V and N, one has the Euler equation

$$N \left.\frac{\partial \mu}{\partial N}\right)_{T,V} + V \left.\frac{\partial \mu}{\partial V}\right)_{T,N} = 0. \tag{3.151}$$

Thus from equations (3.149–150) we obtain

$$k_{\mathrm{B}}T = N \left.\frac{\partial \mu}{\partial N}\right)_{T,V} = -V \left.\frac{\partial \mu}{\partial V}\right)_{T,N} = V \left.\frac{\partial p}{\partial N}\right)_{T,V}, \tag{3.152}$$

where we have exploited a Maxwell relation. Integrating this equation with respect to N, with the obvious boundary condition $p(N = 0) = 0$ yields

$$pV = N\, k_{\mathrm{B}}T. \tag{3.153}$$

Chapter 4. Interaction-Free systems

4.1 Harmonic Oscillators

4.1.1 The Equipartition Theorem

- Page 90, line 16f. Read:
  positive definitive
  correct to:
  positive definite

4.3 Boson and Fermion Gases

4.3.1 Electrons in Metals

- Page 109, line 5, eq. (4.96). Read:

$$C_V = \frac{\pi V^2}{3} k_{\mathrm{B}} T\, \omega(\epsilon_{\mathrm{F}}),$$

  correct to:

$$C_V = \frac{\pi V^2}{3} k_{\mathrm{B}}^2 T\, \omega(\epsilon_{\mathrm{F}}),$$

4.4 Einstein condensation

- Page 114. Caption to figure 4.7, second line. Read:
  the rescaled density $p\lambda^3$.
  correct to:
  the rescaled density $\rho\lambda^3$.

4.5.1 Myoglobin and Hemoglobin

- Page 116, line 6 from bottom. Read:
  $\sum_{\alpha=1}^{N/4} \sum_{i=1}^{4} \tau_{\alpha i}$ of adsorbed molecules
  correct to:
  $\sum_{\alpha=1}^{N/4} \sum_{i=1}^{4} \langle \tau_{\alpha i} \rangle$ of adsorbed molecules

Chapter 6. Renormalization Group

Relevant and Irrelevant Operators

- Page 184, line 10, eq. (6.57). Read:

$$\langle \phi_0 \phi_{\boldsymbol{r}} \rangle_{\mathcal{H}} = b^{2d} \zeta^{-2} \left\langle \phi'_0 \phi'_{\boldsymbol{r}/b} \right\rangle_{\mathcal{H}'} .$$

  correct to:

$$\langle \phi_0 \phi_{\boldsymbol{r}} \rangle_{\mathcal{H}} = b^{-2d} \zeta^{-2} \left\langle \phi'_0 \phi'_{\boldsymbol{r}/b} \right\rangle_{\mathcal{H}'} .$$

- Page 184, line 10, eq. (6.57). Read:

  correct to:

$$d + 2 - \eta = 2\frac{\ln\zeta}{\ln b}.$$

  correct to:

$$d + 2 - \eta = -2\frac{\ln\zeta}{\ln b}.$$

6.6 Renormalization in Fourier Space

6.6.1 Introduction

- Page 190, line 12 from bottom, eq. (6.91). Read:

$$\phi_i = \sum_i \phi_{\boldsymbol{k}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_i}.$$

  correct to:

$$\phi_i = \sum_{\boldsymbol{k}} \phi_{\boldsymbol{k}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_i}.$$

- Page 190, line 6 from bottom. Read:
  For a simple cubic lattice, as we saw in chapter 2, one has
  correct to:
  For a simple cubic lattice, as we saw in chapter 5, one has

6.6.2 Gaussian Model

- Page 193, line 8. Read:
  coefficients of κ^n with $n \neq 0$
  correct to:
  coefficients of k^n in $\Delta(\boldsymbol{k})$ with $n \neq 0$

6.6.4 Critical Exponents to First Order in ϵ

- Page 199, line 5. Read:
  (as we shall from now on)
  correct to:
  (as we shall set from now on)

Chapter 7. Classical Fluids

7.2 Reduced Densities

7.2.3 Measure of $g(r)$

- Page 225, line 7. Read:
  define the **factor structure** $S(\boldsymbol{k})$:

correct to:
define the **structure factor** $S(\boldsymbol{k})$:

7.2.4 BBGKY Hierarchy

- Page 225, line 7 from bottom, Eq. (7.47). Read:

$$\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \rho^{(2)} g(\boldsymbol{x}_1, \boldsymbol{x}_2),$$

correct to:

$$\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \rho^2 \, g(\boldsymbol{x}_1, \boldsymbol{x}_2),$$

7.3 Virial Expansion

- Page 229, line 16. Read:
Exercise 7.2 By comparing (7.70) with (7.12), show that in this approximation the $g(r)$ is expressed by
$$g(r) = 1 + f(r).$$

correct to:
Exercise 7.2 Show that in the present approximation, the $g(r)$ is expressed by
$$g(r) = 1 + f(r),$$
where $f(r)$ is the Mayer function, and that therefore equations (7.12) and (7.70) are compatible.

- Page 230, line 15. Read:
3. Express this quantity as a function of the second virial coefficient $B_2(T)$ and evaluate the inversion temperature T^* in which $\partial T/\partial p)_H$ changes sign.
correct to:
3. By expressing $\partial V/\partial T)_p$ as a function of the second virial coefficient $B_2(T)$, evaluate the inversion temperature T^* in which $\partial T/\partial p)_H$ changes sign.

7.3.1 Higher Virial Coefficients

- Page 235, line 4, eq. (7.96). Read:

$$\frac{p}{pk_\mathrm{B}T} = \frac{1 + \eta + \eta^2 - \eta^3}{(1-\eta)^3}.$$

correct to:

$$\frac{p}{\rho k_\mathrm{B}T} = \frac{1 + \eta + \eta^2 - \eta^3}{(1-\eta)^3}.$$

7.3.4 Convergence of the Fugacity and Virial Expansion

- Page 241, line 11, eq. (7.129). Read:

$$C(T)=\int \mathrm{d}\boldsymbol{x}\,\left(\mathrm{e}^{-u(\boldsymbol{x})/k_{\mathrm{B}}T}-1\right)<+\infty.$$

  correct to:

$$C(T)=\int \mathrm{d}\boldsymbol{x}\,\left|\mathrm{e}^{-u(\boldsymbol{x})/k_{\mathrm{B}}T}-1\right|<+\infty.$$

7.5.1 Ionic Solutions

- Page 248, line 7 from bottom. Read:

$$\frac{E}{T^2}=-\frac{\partial F}{\partial T}$$

  correct to:

$$\frac{E}{T^2}=-\frac{\partial(F/T)}{\partial T},$$

Chapter 8. Numerical Simulation

8.2 Molecular Dynamics

8.2.2 Verlet Algorithm

- Page 258, line 9 from bottom. Read:
  and $\tau=\sqrt{m/\epsilon_0}$ as a time scale,
  correct to:
  and $\tau=\sqrt{mr_0^2/\epsilon_0}$ as a time scale,

8.4 Monte Carlo Method

8.4.1 Markov Chains

- Page 264, line 7. Read:
  if, given any three different states $a,b,c\in Q$, one has

$$W_{ab}W_{bc}W_{ca}=W_{ac}W_{cb}W_{ba}. \tag{8.39}$$

  correct to:
  if, given any k different states $i_1,i_2,\ldots,i_k\in Q$ $(k\geq 3)$, one has

$$W_{i_1i_2}W_{i_2i_3}\cdots W_{i_ki_1}=W_{i_1i_k}W_{i_ki_{k-1}}\cdots W_{i_2i_1}. \tag{8.39}$$

Chapter 9. Dynamics

9.9 Response Functions

- Page 298, line 12. Read:
  $h(t) = h\,\delta(t - t')$.
  correct to:
  $h(t) = h\,\delta(t - t_0)$.

- Page 298, line 13, eq. (9.111). Read:
  $\langle X(t) \rangle = h\,\chi(t, t')$.
  correct to:
  $\langle X(t) \rangle = h\,\chi(t, t_0)$.

- Page 299, line 8 from bottom. Read:
  $x_{ij}(t)$. For $t > 0$,
  correct to:
  $\chi_{ij}(t)$. For $t > 0$,

9.13 Variational Principle

- Page 306, line 1st from bottom. Read:
  an affinity F_i
  correct to:
  an affinity F_1

- Page 307, line 1. Read:
  Then, the result we just obtained that the stationary state...
  correct to:
  Then, the result we just obtained implies that the stationary state...

- Page 307, line 2. Read:
  In fact since...
  ...and we obtain $\partial \dot{S}/\partial X_k = 0$ for $J_k = 0$ $(k \neq 1)$.
  correct to:
  In fact, upon a variation (δF_i) of the forces, we obtain from equation (9.172) the corresponding variation of $\dot{S}$
  $$\delta \dot{S} = \sum_{ij} L_{ij} F_i\, \delta F_j = \sum_j J_j\, \delta F_j, \tag{9.179}$$
  where we have used the relation (9.160). The term with $j = 1$ vanishes because F_1 is kept fixed. Thus $\delta \dot{S} = 0$ implies $J_k = 0$ for $k \neq 1$, since the δF_k with $k \neq 1$ are arbitrary.

Chapter 10. Complex systems

10.2. Percolation

10.2.1 Analogy with Magnetic Phenomena

- Page 322. Eq. (10.47). Read:
  $$\sum_s s\nu_s + P(p) = 1.$$

correct to:

$$\sum_s s\nu_s + P(p) = p.$$

- Page 322, line 3 from bottom. Read:

$$\chi = \frac{J}{k_{\rm B}T}\sum_s s^2\nu_s(p).$$

correct to:

$$\chi = \frac{1}{k_{\rm B}T}\sum_s s^2\nu_s(p).$$

10.2.1 Percolation in One Dimension

- Page 324. Eq. (10.54). Read:

$$S(p) = \frac{\sum_s s^2\nu_s(p)}{\sum_s \nu_s(p)}.$$

correct to:

$$S(p) = \frac{\sum_s s\,\nu_s(p)}{\sum_s \nu_s(p)}.$$

10.2.3 Percolation on the Bethe lattice

- Page 326. Eq. (10.63). Read:

$$S(p) = p\,\frac{1-(\zeta-2)p}{1-(\zeta-1)p}, \qquad \text{for } p < p_{\rm c},$$

correct to:

$$S(p) = p\,\frac{1+p}{1-(\zeta-1)p}, \qquad \text{for } p < p_{\rm c},$$

- Page 327, line 2. Read:
while $\sum_s s\nu_s = 1$,
correct to:
while $\sum_s s\nu_s = p$,

10.3. Disordered systems

10.3.3 Random Energy Model

- Page 344, line 13. Read:
with $\epsilon \ll |E_{\rm c}|$.
correct to:
with $|\epsilon| \ll |E_{\rm c}|$.

10.3.5 The replica method

- Page 349. Eq. (10.168). Read:

$$f = f_0 = k_\mathrm{B}T\ln 2 - \left(\frac{J_0^2}{4k_\mathrm{B}T}\right)$$

  correct to:

$$f = f_0 = -k_\mathrm{B}T\ln 2 - \left(\frac{J_0^2}{4k_\mathrm{B}T}\right)$$

- Page 349, line 2 from bottom. Read:
  The minimum of this free energy is obtained when
  correct to:
  The extremum of this free energy is obtained when

- Page 350. Second line. Add the following sentence after eq. (10.173):
  One may check that the free energy per spin reaches a *maximum*, rather than a minimum, at this value of m. This is just one of the many surprises which appear in the replica method.

Appendix

A. Legendre Transformation

A.2 Properties of the Legendre Transformation

- Page 360, line 9 from bottom. Read:
  Legendre transform g of the f is given by $f = \lambda_1 g+$
  correct to:
  Legendre transform g of the f is given by $g = \lambda_1 g+$

A.3 Legendre Multipliers

- Page 362, line 17 from bottom. Read:
  If we chose λ so that the second member vanishes,
  correct to:
  If we choose λ so that the second term vanishes,

- Page 363, line 4 from bottom. Read:
  with respect to p with ξ fixed
  correct to:
  with respect to ξ with p fixed

B. Saddle Point Method

B.1 Euler Integrals and the Saddle Point Method

- Page 365, line 2, eq. (B5). Read:

$$\lim_{N\to\infty} \frac{I(N)}{\exp[-Nf(x_0)]}\sqrt{\frac{2\pi}{Nf''(x_0)}} = 1.$$

correct to:

- Page 365, line 2, eq. (B5). Read:

$$\lim_{N\to\infty} I(N) \Bigg/ \left(\exp[-Nf(x_0)]\sqrt{\frac{2\pi}{Nf''(x_0)}}\right) = 1.$$

- Page 365, line 7, eq. (B7). Read:

$$\frac{I(N)}{\exp[-Nf(x_0)]}\sqrt{\frac{2\pi}{Nf''(x_0)}} = 1 + \sum_{k=1}^{r}\frac{I_k}{N^k} + \mathrm{o}(N^{-r}).$$

correct to:

$$I(N) \Bigg/ \left(\exp[-Nf(x_0)]\sqrt{\frac{2\pi}{Nf''(x_0)}}\right) = 1 + \sum_{k=1}^{r}\frac{I_k}{N^k} + \mathrm{o}(N^{-r}).$$

- Page 366, line last. Read:
  $f(x)$ admits a maximum
  correct to:
  the expression in brackets in the formula above admits a maximum
- Page 366, line 3. Read:
  integral we studied before, slowly changing factors.
  correct to:
  integral we studied before, up to slowly changing factors.

B3. Properties of N-Dimensional Space

- Page 367, line 8 from bottom, eq. (B.21). Read:

$$\rho(x_1) = \int \prod_{i=2}^{N} \mathrm{d}x_i \; \theta\left(R^2 - x_1^1 - \sum_{i=1}^{N} x_i^2\right).$$

correct to:

$$\rho(x_1) = \int \prod_{i=2}^{N} \mathrm{d}x_i \; \theta\left(R^2 - x_1^1 - \sum_{i=2}^{N} x_i^2\right).$$

Integral Representation of the Delta Function

- Page 368, line 9, eq. (B.25). Read:

$$f(x_0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}y \; \tilde{f}(y) \; \exp\left(\mathrm{i}x_0 y\right).$$

correct to:

$$f(x_0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}y \; \tilde{f}(y) \; \exp\left(-\mathrm{i}x_0 y\right).$$

C. A Probability Refresher

C.2 Random Variables

- Page 370, line 9 from bottom. Read:

$$P(x) = \frac{1}{6} - \sum_{k=1}^{6} \delta(x-k).$$

correct to:

$$P(x) = \frac{1}{6} \sum_{k=1}^{6} \delta(x-k).$$

C.5 Generating Function

- Page 372, line 11 from bottom, eq. (C.20). Read:

$$\langle \exp[\mathrm{i}(x+y)] \rangle = \langle \exp(\mathrm{i}x) \rangle \langle \exp(\mathrm{i}y) \rangle .$$

correct to:

$$\langle \exp[\mathrm{i}k(x+y)] \rangle = \langle \exp(\mathrm{i}kx) \rangle \langle \exp(\mathrm{i}ky) \rangle .$$

- Page 372, line 6 from bottom, eq. (C.21), first line. Read:

$$\langle \exp(\mathrm{i}x) \rangle =$$

correct to:

$$\langle \exp(\mathrm{i}kx) \rangle =$$

C.6 Central Limit Theorem

- Page 373, line 5, eq. (C.23), first line. Read:

$$\langle \exp(\mathrm{i}k\bar{x}) \rangle = \left\langle \exp\left[\mathrm{i}k \frac{1}{N} \left(\sum_{i=1}^{N} \right) \right] \right\rangle = \left\langle \exp\left(\frac{\mathrm{i}kx}{N} \right) \right\rangle$$

correct to:

$$\langle \exp(\mathrm{i}k\bar{x}) \rangle = \left\langle \exp\left[\mathrm{i}k \frac{1}{N} \left(\sum_{i=1}^{N} x_i \right) \right] \right\rangle = \left\langle \exp\left(\frac{\mathrm{i}kx}{N} \right) \right\rangle^N$$

C.7 Correlations

Page 374, line 2. Read:
$\mathsf{Q}^{-1} = (\det \mathsf{A})^{-1/2}$,
correct to:
$\det \mathsf{Q}^{-1} = (\det \mathsf{A})^{-1/2}$,

D. Markov Chains

- `Page 337, line 4.  Read:`
  Let $\nu_k^{(\lambda)}$ be a right eigenvalue of W
  `correct to:`
  Let $\nu_k^{(\lambda)}$ be a right eigenvector of W

E. Fundamental Physical Constants

- `Page 380, line 2.  Read:`
  $\hbar = h/(2/\pi)$
  `correct to:`
  $\hbar = h/(2\pi)$