Errata in "Statistical Mechanics in a Nutshell"

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January 3, 2021

Thanks to L. Buonocore, A. Mehta, M. Sellitto, and especially J. M. Silverman

Chapter 2. Thermodynamics

2.17 Equations of state

- `Page 40, line 12 from bottom. Read:`
  obtained by deriving
  `correct to:`
  obtained by differentiating

Chapter 3. The Fundamental Postulate

3.5 Quantum States

- `Page 66, line 2. Read:`
  moving in one dimension along a segment of length n.
  `correct to:`
  moving in one dimension along a line of length L.

- `Page 66, line 3. Read:`
  possible energy values are $E_n = \hbar^2\pi^2n^2/L^2$
  `correct to:`
  possible energy values are $E_n = \hbar^2\pi^2n^2/(2mL^2)$

3.18 Fluctuations of Uncorrelated Particles

- `Page 88. From the end of line 4 to the end of the section. Read:`
  In fact, one has...
  ...Therefore,

$$p = \frac{k_\mathrm{B}T}{v} = \frac{Nk_\mathrm{B}T}{V}. \tag{3.153}$$

  `correct to:`
  In fact, one has

$$\langle N^2 \rangle - \langle N \rangle^2 = \left.\frac{\partial^2 \ln Z_\mathrm{GC}}{\partial(\mu/k_\mathrm{B}T)^2}\right)_{T,V} = k_\mathrm{B}T \left.\frac{\partial N}{\partial \mu}\right)_{T,V}. \tag{3.150}$$

On the other hand, since μ is an intensive variable, function of T and of the extensive variables V and N, one has the Euler equation

$$N \left.\frac{\partial \mu}{\partial N}\right)_{T,V} + V \left.\frac{\partial \mu}{\partial V}\right)_{T,N} = 0. \tag{3.151}$$

Thus from equations (3.149–150) we obtain

$$k_{\mathrm{B}}T = N \left.\frac{\partial \mu}{\partial N}\right)_{T,V} = -V \left.\frac{\partial \mu}{\partial V}\right)_{T,N} = V \left.\frac{\partial p}{\partial N}\right)_{T,V}, \tag{3.152}$$

where we have exploited a Maxwell relation. Integrating this equation with respect to N, with the obvious boundary condition $p(N = 0) = 0$ yields

$$pV = N\, k_{\mathrm{B}}T. \tag{3.153}$$

Chapter 4. Interaction-Free systems

4.1 Harmonic Oscillators

4.1.1 The Equipartition Theorem

- `Page 90, line 16f. Read:`
  positive definitive
  `correct to:`
  positive definite

4.3 Boson and Fermion Gases

4.3.1 Electrons in Metals

- `Page 109, line 5, eq. (4.96). Read:`

$$C_V = \frac{\pi V^2}{3} k_{\mathrm{B}} T\, \omega(\epsilon_{\mathrm{F}}),$$

  `correct to:`

$$C_V = \frac{\pi V^2}{3} k_{\mathrm{B}}^2 T\, \omega(\epsilon_{\mathrm{F}}),$$

4.4 Einstein condensation

- `Page 114. Caption to figure 4.7, second line. Read:`
  the rescaled density $p\lambda^3$.
  `correct to:`
  the rescaled density $\rho\lambda^3$.

4.5.1 Myoglobin and Hemoglobin

- `Page 116, line 6 from bottom. Read:`
  $\sum_{\alpha=1}^{N/4} \sum_{i=1}^{4} \tau_{\alpha i}$ of adsorbed molecules
  `correct to:`
  $\sum_{\alpha=1}^{N/4} \sum_{i=1}^{4} \langle \tau_{\alpha i} \rangle$ of adsorbed molecules

Chapter 6. Renormalization Group

Relevant and Irrelevant Operators

- Page 184, line 10, eq. (6.57). Read:

$$\langle \phi_0 \phi_{\boldsymbol{r}} \rangle_{\mathcal{H}} = b^{2d} \zeta^{-2} \left\langle \phi_0' \phi_{\boldsymbol{r}/b}' \right\rangle_{\mathcal{H}'}.$$

  correct to:

$$\langle \phi_0 \phi_{\boldsymbol{r}} \rangle_{\mathcal{H}} = b^{-2d} \zeta^{-2} \left\langle \phi_0' \phi_{\boldsymbol{r}/b}' \right\rangle_{\mathcal{H}'}.$$

- Page 184, line 10, eq. (6.57). Read:

  correct to:

$$d + 2 - \eta = 2 \frac{\ln \zeta}{\ln b}.$$

  correct to:

$$d + 2 - \eta = -2 \frac{\ln \zeta}{\ln b}.$$

6.6 Renormalization in Fourier Space

6.6.1 Introduction

- Page 190, line 12 from bottom, eq. (6.91). Read:

$$\phi_i = \sum_i \phi_{\boldsymbol{k}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_i}.$$

  correct to:

$$\phi_i = \sum_{\boldsymbol{k}} \phi_{\boldsymbol{k}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_i}.$$

- Page 190, line 6 from bottom. Read:
  For a simple cubic lattice, as we saw in chapter 2, one has
  correct to:
  For a simple cubic lattice, as we saw in chapter 5, one has

6.6.2 Gaussian Model

- Page 193, line 8. Read:
  coefficients of κ^n with $n \neq 0$
  correct to:
  coefficients of k^n in $\Delta(\boldsymbol{k})$ with $n \neq 0$

6.6.4 Critical Exponents to First Order in ϵ

- Page 199, line 5. Read:
  (as we shall from now on)
  correct to:
  (as we shall set from now on)

Chapter 7. Classical Fluids

7.2 Reduced Densities

7.2.3 Measure of $g(r)$

- Page 225, line 7. Read:
  define the **factor structure** $S(\boldsymbol{k})$:
  correct to:
  define the **structure factor** $S(\boldsymbol{k})$:

7.2.4 BBGKY Hierarchy

- Page 225, line 7 from bottom, Eq. (7.47). Read:

$$\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \rho^{(2)} g(\boldsymbol{x}_1, \boldsymbol{x}_2),$$

  correct to:

$$\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \rho^2 \, g(\boldsymbol{x}_1, \boldsymbol{x}_2),$$

7.3 Virial Expansion

- Page 229, line 16. Read:
  Exercise 7.2 By comparing (7.70) with (7.12), show that in this approximation the $g(r)$ is expressed by

$$g(r) = 1 + f(r).$$

  correct to:
  Exercise 7.2 Show that in the present approximation, the $g(r)$ is expressed by

$$g(r) = 1 + f(r),$$

  where $f(r)$ is the Mayer function, and that therefore equations (7.12) and (7.70) are compatible.

- Page 230, line 15. Read:
  3. Express this quantity as a function of the second virial coefficient $B_2(T)$ and evaluate the inversion temperature T^* in which $\partial T/\partial p)_H$ changes sign.
  correct to:
  3. By expressing $\partial V/\partial T)_p$ as a function of the second virial coefficient $B_2(T)$, evaluate the inversion temperature T^* in which $\partial T/\partial p)_H$ changes sign.

7.3.1 Higher Virial Coefficients

- Page 235, line 4, eq. (7.96). Read:

$$\frac{p}{pk_{\mathrm{B}}T} = \frac{1+\eta+\eta^2-\eta^3}{(1-\eta)^3}.$$

correct to:

$$\frac{p}{\rho k_{\mathrm{B}}T} = \frac{1+\eta+\eta^2-\eta^3}{(1-\eta)^3}.$$

Convergence of the Fugacity and Virial Expansion

- Page 241, line 11, eq. (7.129). Read:

$$C(T) = \int \mathrm{d}\boldsymbol{x} \, \left(\mathrm{e}^{-u(\boldsymbol{x})/k_{\mathrm{B}}T} - 1\right) < +\infty.$$

correct to:

$$C(T) = \int \mathrm{d}\boldsymbol{x} \, \left|\mathrm{e}^{-u(\boldsymbol{x})/k_{\mathrm{B}}T} - 1\right| < +\infty.$$

Chapter 8. Numerical Simulation

8.2 Molecular Dynamics

8.2.2 Verlet Algorithm

- Page 258, line 9 from bottom. Read:
  and $\tau = \sqrt{m/\epsilon_0}$ as a time scale,
  correct to:
  and $\tau = \sqrt{mr_0^2/\epsilon_0}$ as a time scale,

8.4 Monte Carlo Method

8.4.1 Markov Chains

- Page 264, line 7. Read:
  if, given any three different states $a, b, c \in Q$, one has

$$W_{ab}W_{bc}W_{ca} = W_{ac}W_{cb}W_{ba}. \tag{8.39}$$

correct to:
if, given any k different states $i_1, i_2, \ldots, i_k \in Q$ $(k \geq 3)$, one has

$$W_{i_1i_2}W_{i_2i_3}\cdots W_{i_ki_1} = W_{i_1i_k}W_{i_ki_{k-1}}\cdots W_{i_2i_1}. \tag{8.39}$$

Chapter 9. Dynamics

9.9 Response Functions

- Page 298, line 12. Read:
  $h(t) = h\,\delta(t - t')$.
  correct to:
  $h(t) = h\,\delta(t - t_0)$.

- Page 298, line 13, eq. (9.111). Read:
  $\langle X(t)\rangle = h\,\chi(t, t')$.
  correct to:
  $\langle X(t)\rangle = h\,\chi(t, t_0)$.

- Page 299, line 8 from bottom. Read:
  $x_{ij}(t)$. For $t > 0$,
  correct to:
  $\chi_{ij}(t)$. For $t > 0$,

9.13 Variational Principle

- Page 306, line 1st from bottom. Read:
  an affinity F_i
  correct to:
  an affinity F_1

- Page 307, line 1. Read:
  Then, the result we just obtained that the stationary state...
  correct to:
  Then, the result we just obtained implies that the stationary state...

- Page 307, line 2. Read:
  In fact since...
  ...and we obtain $\partial\dot{S}/\partial X_k = 0$ for $J_k = 0$ ($k \neq 1$).
  correct to:
  In fact, upon a variation (δF_i) of the forces, we obtain from equation (9.172) the corresponding variation of $\dot{S}$
  $$\delta\dot{S} = \sum_{ij} L_{ij} F_i\,\delta F_j = \sum_j J_j\,\delta F_j, \tag{9.179}$$
  where we have used the relation (9.160). The term with $j = 1$ vanishes because F_1 is kept fixed. Thus $\delta\dot{S} = 0$ implies $J_k = 0$ for $k \neq 1$, since the δF_k with $k \neq 1$ are arbitrary.

Chapter 10. Complex systems

10.2. Percolation

10.2.1 Analogy with Magnetic Phenomena

- Page 322. Eq. (10.47). Read:
  $$\sum_s s\nu_s + P(p) = 1.$$

correct to:

$$\sum_s s\nu_s + P(p) = p.$$

- Page 322, line 3 from bottom. Read:

$$\chi = \frac{J}{k_{\mathrm{B}}T}\sum_s s^2\nu_s(p).$$

correct to:

$$\chi = \frac{1}{k_{\mathrm{B}}T}\sum_s s^2\nu_s(p).$$

10.2.1 Percolation in One Dimension

- Page 324. Eq. (10.54). Read:

$$S(p) = \frac{\sum_s s^2\nu_s(p)}{\sum_s \nu_s(p)}.$$

correct to:

$$S(p) = \frac{\sum_s s\,\nu_s(p)}{\sum_s \nu_s(p)}.$$

10.2.3 Percolation on the Bethe lattice

- Page 326. Eq. (10.63). Read:

$$S(p) = p\,\frac{1-(\zeta-2)p}{1-(\zeta-1)p}, \qquad \text{for } p < p_{\mathrm{c}},$$

correct to:

$$S(p) = p\,\frac{1+p}{1-(\zeta-1)p}, \qquad \text{for } p < p_{\mathrm{c}},$$

- Page 327, line 2. Read:
while $\sum_s s\nu_s = 1$,
correct to:
while $\sum_s s\nu_s = p$,

10.3. Disordered systems

10.3.3 Random Energy Model

- Page 344, line 13. Read:
with $\epsilon \ll |E_{\mathrm{c}}|$.
correct to:
with $|\epsilon| \ll |E_{\mathrm{c}}|$.

10.3.5 The replica method

- Page 349. Eq. (10.168). Read:

$$f = f_0 = k_B T \ln 2 - \left(\frac{J_0^2}{4k_B T}\right)$$

correct to:

$$f = f_0 = -k_B T \ln 2 - \left(\frac{J_0^2}{4k_B T}\right)$$

- Page 349, line 2 from bottom. Read:
  The minimum of this free energy is obtained when
  correct to:
  The extremum of this free energy is obtained when

- Page 350. Second line. Add the following sentence after eq. (10.173):
  One may check that the free energy per spin reaches a *maximum*, rather than a minimum, at this value of m. This is just one of the many surprises which appear in the replica method.

Appendix

B. Saddle Point Method

0.0.1 B.1 Euler Integrals and the Saddle Point Method

- Page 366, line 3. Read:
  integral we studied before, slowly changing factors.
  correct to:
  integral we studied before, up to slowly changing factors.

C. A Probability Refresher

C.2 Random Variables

- Page 370, line 9 from bottom. Read:

$$P(x) = \frac{1}{6} - \sum_{k=1}^{6} \delta(x-k).$$

correct to:

$$P(x) = \frac{1}{6} \sum_{k=1}^{6} \delta(x-k).$$

C.6 Central Limit Theorem

- Page 373, line 5, eq. (C.23), first line. Read:

$$\langle \exp(\mathrm{i}k\bar{x})\rangle = \left\langle \exp\left[\mathrm{i}k\frac{1}{N}\left(\sum_{i=1}^{N}\right)\right]\right\rangle = \left\langle \exp\left(\frac{\mathrm{i}kx}{N}\right)\right\rangle$$

correct to:

$$\langle \exp(\mathrm{i}k\bar{x}) \rangle = \left\langle \exp\left[\mathrm{i}k\frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)\right]\right\rangle = \left\langle \exp\left(\frac{\mathrm{i}kx}{N}\right)\right\rangle^N$$

D. Markov Chains

- Page 337, line 4. Read:
  Let $\nu_k^{(\lambda)}$ be a right eigenvalue of W
  correct to:
  Let $\nu_k^{(\lambda)}$ be a right eigenvector of W

E. Fundamental Physical Constants

- Page 380, line 2. Read:
  $\hbar = h/(2/\pi)$
  correct to:
  $\hbar = h/(2\pi)$