

## Calculation of Critical Indices.

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(ricevuto il 5 Luglio 1971)

Recent theoretical works <sup>(1)</sup> have shown that the behaviour of propagators near phase transition points may be obtained by solving a system of coupled nonlinear integral equations. The « scaling » hypothesis <sup>(2)</sup> corresponds to neglecting the nonhomogeneous terms. POLYAKOV <sup>(3)</sup> has shown that these equations allow for solutions which are covariant under the conformal group <sup>(4)</sup>. This fixes the form of propagators and three-point vertices up to a multiplicative constant as well as the exponents of two-point correlation functions. We exploit these results in order to obtain closed-form equations in which the only unknowns are the indices.

For definiteness we treat the case of the  $\lambda$ -point of Bose liquids. From conformal group symmetry one obtains

$$(1) \quad \begin{cases} G(\mathbf{x}_1 - \mathbf{x}_2) = g(x_1 - x_2)^{-1-\eta}, \\ D(\mathbf{x}_1 - \mathbf{x}_2) = d(x_1 - x_2)^{-6+2/\nu}, \\ \Gamma(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = c(x_2 - x_3)^{-5+\eta+1/\nu}(x_1 - x_2)^{-1/\nu}(x_1 - x_3)^{-1/\nu}, \end{cases}$$

where  $G$  is the one-particle propagator,  $D$  is the « phonon » propagator and  $\Gamma$  is the three-point vertex. The integral equation for  $\Gamma$  has the form shown in Fig. 1 *a*). Because of conformal symmetry, (1) are the solutions of this equation if the constants  $g$ ,  $d$  and  $c$  are properly chosen. Consequently, one obtains for these constants

$$(2) \quad c = a_1 c(c^2 g^2 d) + a_2 c(c^2 g^2 d)^2 + \dots,$$

<sup>(1)</sup> A. A. MIGDAL: *Sov. Phys. JETP*, **28**, 1036 (1969); A. M. POLYAKOV: *Sov. Phys. JETP*, **28**, 533 (1969).

<sup>(2)</sup> L. P. KADANOFF, W. GÖTZE, D. HANBLÉN, R. HECHT, E. A. S. LEWIS, V. V. PALCIAUSKAS, M. RAYL, J. SWIFT, D. ASPNES and J. KANE: *Rev. Mod. Phys.*, **39**, 395 (1967).

<sup>(3)</sup> A. M. POLYAKOV: *JETP Lett.*, **12**, 381 (1970).

<sup>(4)</sup> G. MACK and A. SALAM: *Ann. of Phys.*, **53**, 174 (1969).

where the  $a_i$ 's are given by explicit calculation of the integrals. Dividing by  $c$  one obtains

$$(3) \quad 1 = a_1 \lambda + a_2 \lambda^2 + \dots,$$

where  $\lambda = c^2 g^2 d$ . We suppose that this equation has a solution with small  $\lambda$ ; this is suggested by the fast convergence of computer calculations. With this hypothesis it should be possible to neglect in (2) all terms but the first.

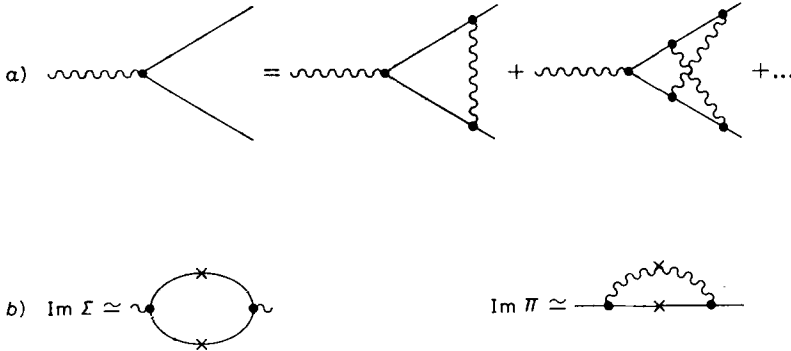


Fig. 1.

The equation for the self-energy part  $\Sigma$  of  $D$  may be obtained from the unitarity condition (1). It is easy to see that the contributions from  $n$ -particle states are proportional at least to  $\lambda^{n-1}$ . It is then possible to neglect all the diagrams but the first. The resulting equation for  $\Sigma$  and the one for the self-energy part  $\Pi$  of  $G$  are shown in Fig. 1 b). We thus obtain the following equations for  $g$ ,  $c$  and  $d$ :

$$(4) \quad g^{-1} = f_1 c^2 g d, \quad d^{-1} = f_2 c^2 g^2,$$

where  $f_1$  and  $f_2$ , which depend on the indices  $\eta$  and  $\nu$ , are obtained by explicitly calculating the integrals. Then (4) and (1) give the equations

$$(5) \quad a_1(\eta, \nu) = f_1(\eta, \nu) = f_2(\eta, \nu)$$

for the indices.

Some of the integrals involved are not well defined for some values of the indices because of spurious ultraviolet divergences. This difficulty may be overcome by defining them in the zone of interest by analytic continuation in  $\eta$  and  $\nu$  from their zone of convergence (\*). Solving eq. (5) the indices are found. Corrections of higher order in  $\lambda$  may be obviously computed with the same method. The integrals defining  $a_1$ ,  $f_1$ ,  $f_2$  are rather awkward; their evaluation is now in progress.

(\*) This tool is well known in analytical renormalization theory (\*).

(\*) E. SPEER: *Generalized Feynman amplitudes*, in *Annals of Mathematical Studies*, No. 62 (Princeton, N. J., 1969).